A COMPARISON OF POINT-SAMPLING WITH PLOT-SAMPLING AT 655 FOREST SURVEY LOCATIONS IN SOUTHEAST TEXAS

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A Comparison of Point-Sampling With
Plot-Sampling at 655 Forest Survey Locations
in Southeast Texas

By

L. R. Grosenbaugh
W. S. Stover

Grosenbaugh is Chief, Division of Forest Management, Southern Forest Experiment Station, Forest Service, U. S. Department of Agriculture. At the time the study was made, Stover was in charge of the Southern Forest Survey, Division of Forest Economics, Southern Forest Experiment Station.

Point-sampling has long been a familiar technique in estimating forest acreage from a map or aerial photo. Recently, through the use of an angle-gauge[1/] which inflates tree basal area on the ground by a

1/ Bitterlich, W. 1948. Die winkelzählprobe. Allgemeine Forst- und Holzwirtschaftliche Zeitung 59 (1/2): 4-5.

constant factor, the point-sampling concept has been extended to estimates of tree volume and tree frequency per acre[2/], as well as to tree basal

2/ Grosenbaugh, L. R. 1952. Plotless timber estimates--new, fast, easy. J. For. 50: 32-37.

area per acre. Although various angle-gauges have been devised, the one with the most promise is a simple wedge-prism[3/], which can be

3/ Müller, G. 1953. Das baumzählrohr. Allgemeine Forstzeitung 64 (19/20): 249-251.

elaborated to allow for slope-compensation and magnification[4/]. The

4/ Bruce, D. 1955. A new way to look at trees. J. For. 53:163-167.

development and application of point-sampling theory as applied to trees has been summarized in a recent publication[5/].

5/ Grosenbaugh, L. R. 1955. Better diagnosis and prescription in southern forest management. U. S. Forest Serv. South. Forest Exp. Sta. Occas. Paper 145, 27 pp.

Because of the fundamental difference between plot-sampling (where probability of tree selection is proportional to tree frequency) and point-sampling (where probability of tree selection is proportional to tree basal area), many people have tried to evaluate point-sampling by comparing it with complete enumeration on a small tract. Often such comparisons have been vitiated by improper calibration of the angle-gauge, use of volume/basal area ratios not based on the same tree volume figures that were used in complete enumeration, brush bias, edge-effect bias, biased treatment of doubtful trees, bias in locating sample points, misinterpretation of normal variation attributable to sampling, etc.

Sample Design and Field Procedures

The initiation of the U. S. Forest Service Forest Survey in 12 counties of southeast Texas in 1954 presented the Southern Forest Experiment Station with an excellent opportunity of making a valid comparison of plot-sampling with point-sampling, free from any of the above-mentioned sources of erroneous inference.

The counties were to be inventoried by standard Forest Survey plot procedure. Since in this area slope rarely exceeded 10 percent (at which a 1-percent correction factor is needed in point-sampling), it was decided to also select sample trees with a simple 104.18-minute wedge-prism (3.03 prism-diopters) by observation from the center of each 1/4-acre circular plot on which regular Forest Survey data were taken. Both sets of data would later be processed by punched card techniques; then basal area, cubic-foot, and board-foot density figures for each plot and concentric point-sample could be compared and analyzed. Since field crews could not work up tally sheets manually, they were unaware of the outcome of the check until long after field work was completed.

Essentially, Forest Survey procedure in southeast Texas involved obtaining a pair of 1/4-acre plots (5-chain separation) at each intersection of a square grid (3-mile separation) falling on forested land. Although full Forest Survey data were taken on both plots (known as the principal plot and the auxiliary plot), the bulk of the comparisons and analyses discussed below are based on only the 655 principal plots obtained, with their concentric point-samples, in the 12 Texas counties inventoried in 1954-1955. Basal areas on the 655 auxiliary points were worked up for a special analysis, however, so that an estimate of "within location" and "between location" variances could furnish a clue to possible changes in design of future surveys using point-samples.

Rather than have several crews use wedge-prisms with basal-area factors which differed, it was considered preferable to procure a large number of commercial 3-diopter wedge-prisms and select only those whose deviation as measured by several people and with two target sizes (3.03 inches and 9.09 inches) averaged 104.18 minutes or 3.03 diopters. Calibration (except that two target sizes were used) followed the procedure outlined in[5/] cited earlier. Any prism whose mean basal area factor lay between 9.9 and 10.1 was accepted, and all others were discarded. In effect, this allowed the basal area factor 10.0 to be used for every prism, with less than 1 percent possible error. Three prisms meeting the above standards were found out of a dozen 3-diopter prisms calibrated. The "doubtful tree" check described below practically eliminated any remaining instrumental error, and also removed personal bias when borderline trees were encountered.

Any tree viewed through the prism which could not be clearly classed as "tally" (overlapping image) or "non-tally" (separated image) was checked by measurement. If distance to the tree (in feet and tenths) was less than 2.75 times tree d.b.h. (in inches and tenths), the doubtful tree was tallied. This 2.75 was the "plot radius factor" appropriate to the prism used; theory and application are discussed in[2/] and[5/] cited earlier.

On the Survey, detailed sample-tree measurements (to the nearest tenth-inch of diameter) were taken for all trees lying in the quarter-acre plot and having d. b. h. larger than 6.95 inches. On a concentric 1/40-acre plot, trees with d. b. h. from 4.95 to 6.95 were measured to the nearest tenth-inch and trees with d. b. h. in the 2-inch intervals from .95 to 2.95 inches, and from 2.95 to 4.95 inches were tallied by frequency only (these will henceforth be referred to as 1 to 3 and 3 to 5 inch classes). However, the point-sample included all trees larger than 1 inch in diameter. In order to remove as much bias as possible from the 1/40-acre plot-estimate of the basal area in small trees in the nominal 2-inch and 4-inch classes, the senior author devised a technique using the ratio of two integrals (that of the basal area distribution over that of the frequency distribution). The technique is described in detail in the Appendix, and resulted in lowering the plot-estimate of basal area per acre nearly 1 square foot, as compared to what would have been obtained by the conventional biased technique of assuming that the basal area of the tree at the class mid-diameter is the same as the tree of mean basal area. This indicates the magnitude of bias which may be injected by neglecting grouping error in basal-area estimates.

Gross cubic-foot volumes (excluding bark but including cull) to a merchantable top diameter of not less than 4.0 inches inside bark were calculated for all sampled trees with a d. b. h. larger than 4.95 inches. Standard Forest Survey volume tables (entered with d. b. h. to nearest inch, merchantable height to nearest 8 feet, and Girard form-class) were used for plot-sampled trees, and the same volume tables were used to derive volume/basal area ratios for point-sampled trees.

Gross board-foot volumes (excluding bark but including cull) to a merchantable top diameter (not less than 6.0 inches inside bark for conifers and 8.0 inches for others) were calculated for all conifers with d. b. h. larger than 8.95 inches and all hardwoods with d. b. h. of more than 10.95 inches. Standard Forest Survey volume tables (International log rule, with 1/4-inch kerf) were used with the same entering variables as in the case of cubic feet. Ratios of board-foot volume to basal area were also derived from point-sampled trees as has been discussed above.

Results

In general, the comparison of point-sampling estimates with plot-sampling estimates substantiated what might have been expected from any well-conducted comparison of two unbiased sampling methods. No greater differences in mean basal area, cubic-foot volume, or board-foot volume per acre were observed than those which might easily be attributed to chance or sampling variation. Although sampling with a systematic square grid complicates estimates of error, random-sampling formulae have been employed since evidence seems to indicate that errors thus calculated will be on the conservative side.

As can be seen from the per-acre comparisons in Table 1, the point-sample estimate was .041 sq. ft. per acre lower in basal area, .4 cubic feet per acre higher in cubic-foot volume, and 37 board feet per acre lower in board-foot volume than the plot-sample estimate, where there were 655 points each in the center of the same number of 1/4-acre plots scattered all over southeast Texas. When compared with appropriate standard errors of the difference, none of these approached the 5 percent level of significance--indeed, larger differences would have arisen more than half the time merely from sampling variation. Table 1 also gives standard deviations and standard errors for each type of sample--although the point-sample errors were slightly higher, the work involved was far less. Comparative times needed by each technique were not obtained in

Table 1 Comparison of means, standard de[illegible]ions, and standard errors obtained for severa[illegible]riables on 655 forested locations in southeast Texas point-sampled by 104.18 minute prism (Method A) at center of standard one-quarter-acre circular plot (Method B)

Basal-Area Comparison

Sampling technique	Basal area per acre: All trees larger than 1 inch d.b.h.			Probability that greater difference in means might occur by chance in sampling zero-difference populations
	Mean	St. error	St. dev.	
	- - - - Sq. ft. - - - -			
Point-sample (A)	53.76	1.49	38.2	$\frac{.041}{.727} = .056 = t_{.955}$
Plot-sample (B)	53.80	1.25	32.0	95 percent probability
Difference (A-B)	-.041	.727	18.6	

Cubic-Volume Comparison

	Gross cu. vol. per acre: All trees larger than 5 inches d.b.h.			Probability that greater difference in means might occur by chance in sampling zero-difference populations
	Mean	St. error	St. dev.	
	- - - - Cu. ft. - - - -			
Point-sample (A)	692.1	25.9	662.9	$\frac{.4}{10.7} = .037 = t_{.970}$
Plot-sample (B)	691.7	23.6	603.4	97 percent probability
Difference (A-B)	+.4	10.7	273.3	

Board-Foot Comparison

	Gross board foot volume per acre: All trees larger than 11 inches d.b.h. plus 9-11 inch conifers			Probability that greater difference in means might occur by chance in sampling zero-difference populations
	Mean	St. error	St. dev.	
	- - - - - - Bd. ft. - - - - - -			
Point-sample (A)	3260	150.	3848	$\frac{37.}{59.4} = .622 = t_{.534}$
Plot-sample (B)	3297	143.	3663	53 percent probability
Difference (A-B)	-37	59.4	1522	

this preliminary test, so the relative efficiency of the two methods cannot be stated precisely. There is little doubt,however, that an angle-gauge tally is more efficient than the quarter-acre plot tally since the angle-gauge reduced the sampling intensity for small trees while increasing that for trees larger than 21.4 inches in d.b.h., whose volume is important. Possibly an angle-gauge with a factor higher than 10 may prove more efficient under certain conditions; no attempt was made to study this, since it can be ascertained by far less expensive methods than those employing regular Forest Survey crews.

Table 2 gives the same standard deviations and standard errors shown in Table 1, but expresses them as a percent of the mean (in the case of the standard deviation of the difference, it is expressed as a percent of the geometric mean). Here it is apparent that the differences indicated in Table 1 and mentioned above are very small--point-sampling was only 0.1 percent lower in basal area per acre, 0.1 percent higher in cubic-foot volume per acre, and 1.1 percent lower in board-foot volume. The coefficients of variation for point-sampling were slightly higher than those for quarter-acre plot-sampling--from 7 to 12 percent higher depending on whether board feet, cubic feet, or basal area are involved. Standard errors by either method were within 0.5 percent of each other.

Table 2. Comparison of coefficients of variation and correlation, actual percentage differences, and percentage standard errors for several east Texas variables point-sampled by 104.18 minute prism (method A) at center of standard one-quarter acre circular plot (Method B)

Variables sampled	Coefficient of variation for (A): Point method	Coefficient of variation for (B): Plot method	Coefficient of variation for (A – B): Difference (as percent of $\sqrt{AB}$)	Coefficient of correlation between plot and point	Actual difference: $\frac{\text{mean A}}{\text{mean B}} - 1$	Standard error of * $\frac{\text{mean A}}{\text{mean B}}$	Standard error of mean A: Point method	Standard error of mean B: Plot method
	- - Percent - -				- - - - - - Percent - - - - - -			
Basal area	71	59	35	+ .874	–.1	$\pm$ 1.4	$\pm$ 2.8	$\pm$ 2.3
Cubic volume	96	87	40	+ .911	+.1	$\pm$ 1.6	$\pm$ 3.7	$\pm$ 3.4
Bd. ft. volume	118	111	46	+ .919	–1.1	$\pm$ 1.8	$\pm$ 4.6	$\pm$ 4.3

* The standard error of a ratio of means can be approximated in percent as:

$$\frac{C_{A-B}}{\sqrt{N}} = \sqrt{\frac{C^2_A + C^2_B - 2r_{AB}C_AC_B}{N}}$$

where C_A and C_B are coefficients of variation, where C_{A-B} is the standard deviation of the difference between A and B, expressed as a percent of their geometric mean, and where r_{AB} is the correlation coefficient.

It is interesting to note the correlation between the quarter-acre plot observations and the concentric 104.18-minute point-sample observations. Correlation coefficients are .874, .911, and .919 for basal area, cubic-foot volume, and board-foot volume respectively. Although these are relatively high, as might be expected, the difference between an individual plot and its concentric point may exceed 35, 40, or 46 percent of plot estimates about 1 time out of 3 (see coefficient of variation of difference). The coefficients of variation and correlation can be related to one another and to the standard error of the ratio $\frac{\text{point mean}}{\text{plot mean}}$ by the formula set forth at the bottom of table 2.

Before discussing frequency tables and comparisons of the methods at various density levels, it seems desirable to bring out certain underlying facts about 1/4- and 1/40-acre plots and concentric 104.18-minute point-samples. Where a 104.18-minute angle-gauge is used and all trees are exactly 21.4 inches in d.b.h., it is equivalent to taking a 1/4-acre plot (if all trees were exactly 6.8 inches in d.b.h., it would be equivalent to taking a 1/40-acre plot). Thus where trees tend to an average size smaller than 21.4 inches in d.b.h. (as they usually do in southeast Texas), the angle-gauge will tend to sample only some interior part of the 1/4-acre plot, even though it may tally an occasional tree larger than 21.4 inches outside the 1/4-acre plot. Similarly, the 104.18-minute point-sample will always tally 1- to 5-inch trees on some interior part of the 1/40-acre plot.

The situation, then, is analagous to enlarging a 1/40-acre plot to a concentric 1/4-acre plot. Even though the average of the two estimates tends to be the same, extremes will occur much more frequently in the small plot. This is due to the patchy, clustered, or contagious nature of the density distribution. When a very high density is found on the small plot, enlarging the plot usually results in inclusion of density more nearly average. When a very low density is found on the small plot, enlarging it usually will also tend to include density more nearly average. Enlarging a small plot of nearly average density will tend to include areas of higher or lower density in more nearly equal proportions. As a rule, then, extremely high or low densities on small plots tend to regress towards the population mean when the plot is expanded to larger size. Even though the frequency distribution of large-plot estimates of small plots is skewed at both high and low densities of small plots, both large and small plots furnish unbiased estimates of the population mean.

On the other hand, when a very high density is found on a large plot, a sample of a small part of it will not be in any way affected by the population mean outside the large plot. Instead, the small sample will tend to fall higher or lower than the large plot mean with nearly equal frequency (unless there is a badly skewed density distribution on the large plot). When a very low density (near but not quite zero) is found on a large plot, distribution is inevitably skewed, so a small plot will sample zero frequently, but occasionally it will hit a concentration much higher than the plot average. On the whole, the small plot tends to give an unbiased estimate of the large plot, and both tend to give an unbiased estimate of the population, as is well known.

A glance at table 3 will indicate the pertinence of the above discussion, if point-sampling be regarded as the small-plot estimate of the 1/4-acre plot. If a vertical column of frequencies is viewed, it illustrates what happens when point-samples (or small plots) are compared with their enlargement (1/4-acre plots). When the points sample low density, the expanded plots tend to be higher. If the points sample high density, the expanded plots tend to be lower. This is not bias, since the mean of the entire range of points tends to coincide with the population mean--it merely indicates that extremes of high or low density are much more frequent with points than with plots. The plot means are pulled toward the population mean more strongly than are the point means.

Table 3.—Frequency correlation table showing relationship of 655 plot-sample estimates to associated point-sample estimates of timber in east Texas.

Quantity sampled	Class limits for plot estimates	Square feet of basal area per acre: 0-20	20-40	40-60	60-80	80-100	100-120	120-140	140-160	160-180	180-200	200-220	220-240	240-260	Totals
		Frequency of occurrence													
Basal area	0-20	74	12												86
	20-40	25	72	40	10										147
	40-60	4	32	75	37	11									159
	60-80	2	9	17	52	41	12	4	1						138
	80-100			4	15	23	24	9	1						76
	100-120			1	2	8	5	7	5	1					29
	120-140						1	4	5	2	1				13
	140-160						1		1	1		1	1		5
	160-180														
	180-200											1			1
	200-220														
	220-240														
	240-260												1		1
	Total	105	125	137	116	83	43	24	13	4	1	2	2		655

Quantity sampled	Class limits for plot estimates	Gross cubic feet per acre: 0-400	400-800	800-1200	1200-1600	1600-2000	2000-2400	2400-2800	2800-3200	3200-3600	3600-4000	4000-4400	Totals
		Frequency of occurrence											
Gross cubic foot volume	0-400	223	33	4									260
	400-800	47	104	28	3								182
	800-1200	4	27	45	29	5							110
	1200-1600	1	1	13	21	10	5						51
	1600-2000		2	2	5	7	7	3	2				28
	2000-2400			1	2	3	3	2	1				12
	2400-2800						1	3	2				6
	2800-3200								1	1			2
	3200-3600						1						1
	3600-4000									1	1		2
	4000-4400											1	1
	Total	275	167	93	60	25	17	8	6	2	1	1	655

Quantity sampled	Class limits for plot estimates	Gross M board feet per acre: 0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-24	24-26	26-28	Totals
		Frequency of occurrence														
Gross M board foot volume	0-2	266	41	4												311
	2-4	53	73	29	4											159
	4-6	4	17	33	19	7										80
	6-8	1	1	8	17	11	1									39
	8-10	1	1	6	5	7	3	2	1	1						27
	10-12				2	6	3	3		2						16
	12-14				1	2	2		2	1						8
	14-16							3	3	3						9
	16-18															
	18-20							1			1	1				3
	20-22															
	22-24											1				1
	24-26														1	1
	26-28													1		1
	Total	325	133	80	48	33	9	9	6	7	1	2		1	1	655

Now if a horizontal row of frequencies in table 3 is viewed, it illustrates what happens when 1/4-acre plots are sampled by points (or smaller plots). It will be noted that there is much less skew, except at the very lowest plot densities. In general, the point means are pulled toward the plot means, and are not much affected by the population mean.

Table 4 summarizes the plot frequency data horizontally from table 3. It also provides comparisons of point and plot estimates of basal area, cubic-foot volume, and board-foot volume per acre by plot density classes (in this respect it merely breaks down the totals given in table 1). Mean differences are tabulated by plot density class to show that they tend to be small when samples are numerous, and that their sign is not notably correlated with stand density.

In addition to this, table 4 throws some light on how differences between point and plot estimates or ratios of $\frac{\text{point estimate}}{\text{plot estimate}}$ behave. It is noticeable that the variance of individual differences about zero tends to increase roughly as plot estimates increase, and that the variance of individual ratios about unity tends to decrease roughly as plot estimates increase (as long as at least 20 observations are available in a class). This is helpful in substantiating the hypothesis that $\frac{\Sigma \text{ point estimates}}{\Sigma \text{ plot estimates}}$ (or its reciprocal) would have furnished the best estimate of a correction ratio in case an instrumental or personal bias had caused the ratio to differ noticeably from 1.

Table 4.—Point-sample and plot-sample differences in means and ratios in east Texas, broken down by plot-density class to show behavior of variances about difference of zero and ratio of unity; n degrees of freedom rather than n-1 are employed.

QUANTITY SAMPLED	CLASS LIMITS FOR PLOT ESTIMATES PER ACRE	PLOT FRE-QUEN-CY	MEAN DENSITY PER ACRE		DIFFERENCE IN MEAN DENSITY: POINT-PLOT	VARIANCE OF INDIVIDUAL DIFFERENCE ABOUT ZERO	RATIO OF MEANS: POINT / PLOT	UNWEIGHTED VARIANCE OF INDIVIDUAL RATIO ABOUT UNITY
			POINT	PLOT				
	Sq. Ft.	No.	- - - -	- - - -	Sq. Ft. - - -	- - - - - -		
	0-20	86	6.5	8.4	-1.9	44	.774	.888
	20-40	147	30.0	30.5	-.5	199	.984	.216
	40-60	159	46.9	49.1	-2.2	308	.955	.129
	60-80	138	70.4	68.8	+1.6	494	1.023	.102
	80-100	76	90.1	87.6	+2.5	489	1.029	.066
BASAL AREA	100-120	29	108.3	109.3	-1.0	770	.991	.064
	120-140	13	139.2	132.1	+7.1	469	1.054	.027
	140-160	5	168.0	148.0	+20.0	1,959	1.135	.084
	160-180							
	180-200	1	200.0	186.0	+14.0	196	1.075	.006
	200-220							
	220-240							
	240-260	1	230.0	257.0	-27.0	729	.895	.011
	0-260	655	53.8	53.8	0.0	346	.999	.230
	Cu. Ft.	No.	- - - -	- - - - -	Cu. Ft. - -	- - - - - - -		
	0-400	260	203.3	210.8	-7.5	18,457	.964	.681
	400-800	182	574.8	584.4	-9.6	50,751	.984	.147
	800-1200	110	1,006.3	980.7	+25.6	97,062	1.026	.102
GROSS CUBIC-FOOT VOLUME	1200-1600	51	1,423.6	1,381.5	+42.1	154,980	1.030	.081
	1600-2000	28	1,862.9	1,791.0	+71.9	370,202	1.040	.120
	2000-2400	12	1,984.8	2,198.3	-213.5	291,979	.903	.064
	2400-2800	6	2,728.7	2,612.7	+116.0	86,405	1.044	.013
	2800-3200	2	3,226.0	3,133.0	+93.0	75,730	1.030	.005
	3200-3600	1	2,173.0	3,440.0	-1267.0	1,605,289	.632	.140
	3600-4000	2	3,699.5	3,724.0	-24.5	36,510	.993	.000
	4000-4400	1	4,177.0	4,178.0	-1.0	1	1.000	.000
	0-4400	655	692.1	691.7	+.4	74,555	1.001	.341
	MBM	No.	- - - - - -	- - Bd. Ft.	- - - - - - - -	MBM		
	0-2	311	814	826	-12	471	.985	.674
	2-4	159	2,841	2,839	+2	1,724	1.001	.215
	4-6	80	5,074	5,047	+27	3,507	1.005	.147
	6-8	39	7,011	6,784	+227	3,714	1.033	.079
	8-10	27	8,341	8,915	-574	11,954	.936	.147
	10-12	16	10,940	10,758	+182	7,798	1.017	.066
GROSS BOARD-FOOT VOLUME	12-14	8	11,435	12,894	-1,459	13,718	.887	.084
	14-16	9	15,057	14,950	+107	4,041	1.007	.019
	16-18							
	18-20	3	17,242	19,345	-2,103	22,809	.891	.060
	20-22							
	22-24	1	20,613	22,004	-1,391	1,935	.937	.004
	24-26	1	26,072	25,336	+736	542	1.029	.001
	26-28	1	25,084	27,192	-2,108	4,444	.922	.006
	0-28	655	3,260	3,297	-37	2,313	.989	.404

As is well known, $\frac{\Sigma XY}{\Sigma X^2}$ is a valid correction ratio where variance of individual ratios $\frac{Y}{X}$ varies inversely as X^2, and $\frac{\Sigma Y}{\Sigma X}$ is a valid correction ratio where variance of individual ratios varies inversely as X, and $\frac{1}{n}\sum^{n}\frac{Y}{X}$ is a valid correction ratio where variance of individual ratios is homogeneous throughout the range of X. Since the ratios $\frac{\Sigma \text{ point estimates}}{\Sigma \text{ plot estimates}}$ for the 655 locations (basal area or volume) were close enough to 1 for the deviation to be attributable to chance, no correction factor was needed here. Had less rigorous calibration or less precise treatment of doubtful trees been employed, however, correction factors derived in this way would have been useful in assessing and correcting bias from either source.

One interesting fact is apparent from table 4. The only individual class ratio whose deviation from 1 does not appear attributable to chance is in the 0-20 sq. ft. basal area class; the ratio of .774 shown for 86 locations (when taken in conjunction with the .964 cubic-foot ratio and the .985 board-foot ratio in the lowest volume classes) indicates either a slight tendency of the cruisers to avoid thickets of trees smaller than 5 inches in locating plot centers, or else that the high bias in the plot tally caused by 2-inch class grouping of trees smaller than 5 inches was not completely removed even by the improved technique described in the Appendix. Whatever the explanation, basal-area difference in this lowest density class could affect the mean per acre less than 1/2 of 1 percent.

Additional Analyses

In order to obtain variances needed to estimate the optimum number of points per cluster, point-sampling estimates of basal area per acre were made at both principal and auxiliary points at each location.

A tabular comparison of point-sampled basal area statistics at the 655 principal points (1) and their 655 associated auxiliary points (2) is of interest. Principal and auxiliary points were 5 chains apart.

Mean basal area per acre		Coefficients of variation			Coefficient of correlation
				Diff. as percent of $\sqrt{A_1 A_2}$	
A_1	A_2	C_1	C_2	$C_{(1-2)}$	r_{12}
- - Sq. ft. - -		- - -Percent- - -			
53.8	53.2	71	74	78	.418

It can be seen that the variation between two forested points separated by 5 chains (78 percent) is much less than that between two forested points separated by 3 miles ($\sqrt{71^2 + 74^2} = 102$ percent). It is also interesting to note that the correlation coefficient of .418 for points separated by 5 chains is much lower that that of .874 shown in table 2 for concentric points and plots (as might be expected).

The analysis of variance for the 655 pairs of points was:

Source of variation	dfs	ss	ms
Between pairs (location)	654	13,966.1	21.355
Within pairs (points)	655	5,737.5	$8.760 = V_w$
	1309	19,703.6	$\frac{12.595}{2} = 6.298 = V_B$

The expense of point-sampling within locations and between locations (3-mile grid) can be roughly estimated as follows, using approximate car operation costs of $2.00 per moving hour, and 2-man crew costs of $4.25 per hour whether on plot, travelling afoot, or in car:

Cluster expense (per location): (between clusters)	.97 hours crew-plus-car travel	at $6.25 =	$6.06
	.46 hours crew foot-travel	at $4.25 =	1.96
Total cost per location			$\$8.02 = E_B$
Element expense (per point): (within clusters)	.30 hours crew tally-time		
	.11 hours crew foot-travel		
	.41 hours crew	at $4.25 =	$\$1.74 = E_w$

The above estimates of time do not include normal delays or time lost because of weather, equipment breakdown, leave, etc.

With an estimate of variance between and within locations (V_B and V_w), and cost between and within locations or clusters (E_B and E_w), a cluster-sampling formula can be used to approximate the optimum number of points per cluster for greatest efficiency:

$$\text{Points per cluster} = \sqrt{\frac{V_w}{V_B} \cdot \frac{E_B}{E_w}} = \sqrt{\frac{8.760}{6.298} \cdot \frac{8.02}{1.74}}$$

$$= 1.18\sqrt{4.61}$$

$$= (1.18)(2.15)$$

$$= 2.54 \text{ points per location or cluster}$$

Thus, if the present system of a 3-mile grid were to be continued using point-sampling, and if the above costs and variances were valid, it would be slightly more efficient to take 3 points per location instead of the 2 actually taken.

One of the important advantages of the point-sample method is that it permits one man to work alone efficiently. Compared with a two-man crew, one man will need somewhat more tally time at a point, but the cost of travel per location is greatly reduced. Using the above car operation cost and a 1-man crew cost of $2.25 per hour, and estimating that a 1-man crew will require 10 percent more travel time by car and foot and 0.50 hours tally time at each point, cluster expense would be $5.67 per location and element expense $1.39. For 2-point locations the 1-man crew cost would total $8.45 vs. the 2-man crew cost of $11.50, a reduction in cost of 26 percent per location.

Because of difficulty of terrain and safety hazards it is not always practical for one man to work alone. To the extent that conditions permit, however, use of one-man crews offers the opportunity to reduce cost substantially.

Obviously, other modifications would further change the picture. A different grid-spacing, a different size of angle-gauge, the use of 1-man instead of 2-man crews, and possibly the use of helicopters instead of cars would modify the optimum number of points per cluster. It is also possible that the ratio of variance within cluster to that between clusters might be slightly different for volume than for basal area.

Conclusions

This exploratory study demonstrated how comparisons between point-sampling and plot-sampling should be analyzed. Point-sampling estimates of basal area and volume per acre were found to be unbiased with respect to plot-sampling estimates when precautions were observed as described in [5]/. Coefficients of correlation and variation in southeast Texas may not be valid for other areas, but they are at least indicative of the comparative magnitudes and trends apt to be exhibited elsewhere, and may be useful in sampling design. A rough estimate of optimum cluster-size indicated that 3 points per cluster would have been slightly better than 2, but this conclusion might be modified if crew, grid, gauge, or conveyance is changed. Finally, a technique was developed for reducing bias in basal-area estimates when tree frequency only has been tallied by rather broad d. b. h. classes (2 inches or more).

Appendix

Although the point-sample included all qualified trees larger than 1 inch in d. b. h., the plot-sample excluded trees less than 4.95 inches in d. b. h. from the 1/4-acre plot tally. Frequency of trees whose d. b. h. was 1.00 to 3.00 inches and 3.00 to 4.95 inches was sampled only on a concentric 1/40-acre plot, and diameter was recorded by two-inch classes rather than the tenth-inch classes used for trees larger than 4.95 inches. To permit accurate calculation of the basal area per acre for 1- to 5-inch trees on the plot-sample, a new technique was worked out which recognized the effect of J-shaped small-tree frequency distributions on mean tree basal area. This new method is described below, since it is much less biased than the commonly used method which assumes that all trees are clustered at the nominal mid-point of the broad diameter class.

A convenient J-shaped distribution commonly encountered in forests is that popularly known as de Liocourt's (with integrals in geometric progression, so that on semilog paper the logarithmic frequency ordinate plots as a straight line over the arithmetic abscissa, which is diameter). The generating function of the distribution is $y = ke^{-ax}$, where (y) is an element of frequency and (x) is diameter; (a) and (k) are parameters which can be empirically deduced from actual stand table data, although (k) is never explicitly needed.

The frequency per acre (f) of trees in the diameter interval (d) to (d + m) is:

$$f = \int_{d}^{d+m} Y dx = \frac{k e^{-ad}}{a}\left(1 - e^{-am}\right)$$

when (m) denotes a constant width of diameter class, the ratio of frequencies in two adjacent diameter classes is:

$$\frac{f'}{f} = \frac{\int_{d+m}^{d+2m} Y dx}{\int_{d}^{d+m} Y dx} = e^{-am}$$

From the above identities, it is apparent that:

$a = \frac{-2.3026}{m} \log_{10}\left(\frac{f'}{f}\right)$. Calculation of (k) is not necessary. **Mean** basal area per tree in an interval (d) to (d + m) can be calculated when $\frac{f'}{f}$, (a), (d), and (m) are known.

It is $\dfrac{.005454 \int_{d}^{d+m} X^2 Y dx}{\int_{d}^{d+m} Y dx}$, which can be evaluated as follows:

$$\left[\frac{.005454}{a^2\left(1 - \frac{f'}{f}\right)}\right]\left[(ad+1)^2 + 1 - \frac{f'}{f}\left\{\left[(a)(d+m)+1\right]^2 + 1\right\}\right]$$

Multiplying this by (f) for the class in question will, of course, give total basal area in the desired diameter interval.

The 1/40-acre plot-samples on each 1/4-acre plot indicated that there were 224.3 trees per acre between 1 and 3 inches in diameter (called two-inch trees), and 73.7 trees per acre between 3 and 5 inches in diameter (called four-inch trees). Thus $\frac{f'}{f}$ = .3285 where diameter interval (m) = 2 inches. Fitting the distribution previously discussed indicated that a = .5565. The basal area of the mean tree in the class interval 1 to 3 inches was .0198 square feet (a tree with d.b.h. = 1.89 inches), and in the class interval 3 to 5 inches was .0812 square feet (a tree with d.b.h. = 3.85 inches). The basal area of the mean tree for the 2 classes combined (i.e., the interval 1 to 5 inches) was .0350 square feet (a tree with d.b.h. = 2.53 inches).

The tabular comparison in table 5 indicates that use of mid-class tree basal area for each 2-inch class would have resulted in a high bias of 8-1/2 percent (about 1 square foot per acre). If the tree at the nominal midpoint (3 inches) of a single 4-inch class (1-5 inches) had been used, the high bias would have been more than 40 percent (over 4 square feet per acre).

Table 5. Comparison of mid-class and mean-tree methods of computing basal area in 2-inch and in 4-inch diameter classes on plots in east Texas

Class limits	Number of trees per acre	Mid-class method		Mean-tree method		Basal area per acre			
						Mid-class method		Mean-tree method	
		Mid-class tree d. b. h.	BA of mid-class tree	Mean tree d. b. h.	BA of mean tree	Two 2" classes	Single 4" class	Two 2" classes	Single 4" class
Inches		Inches	Sq. ft.	Inches	Sq. ft.				
1-3	224.3	2	.0218	1.89	.0198	4.89		4.44	
3-5	73.7	4	.0873	3.85	.0812	6.43		5.99	
1-5	298.0	3	.0491	2.53	.0350	11.32	(14.63)	10.43	(10.43)

Even the 1 square foot basal area bias shown above for midclass trees of each 2-inch class/would have vitiated comparability of plot-sample means with point-sample means, but the mean-tree method described above made use of observed $\frac{f'}{f}$ to take into account the J-shape of the frequency curve and minimize the bias.

Although extrapolation is usually undesirable, the above technique could be used in all-aged stands to extrapolate frequencies and basal areas above or below the sampling limits. Non-observed f'' for the larger diameter class from (d +2m) to (d + 3m) would be calculated as:

$$f'' = f'\left(\frac{f'}{f}\right)$$

Mean basal area per tree for these f'' trees could be calculated by evaluating the appropriate integral as explained earlier.